Quantum logic gates with controllable and selective interaction for superconducting charge qubits via a nanomechanical resonator

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In this paper, we propose a scheme to implement two-qubit logic gates with a controllable and selective interaction in a scalable superconducting circuit of charge qubits. A nanomechanical resonator is used as a data bus to connect qubits. It is indicated that a controllable interaction between qubits can be obtained by making use of the data bus. It is shown that a selective interaction between qubits can be realized when many qubits are involved in the system under our consideration.

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Recently, much attention has been paid to superconducting quantum circuits (SQCs) due to their potential applications in quantum information processing [1, 2, 3]. In SQCs there are usually three types of qubits, i.e., charge qubits [4, 5, 6, 7, 8], phase qubits [9, 10, 11, 12, 13], and flux qubits [14, 15, 16, 17, 18, 19]. As solid systems, SQCs have the advantage of being able to be integrated to a large scale.

In order to realize quantum computation, one need a universal quantum logic gate set which consists of single-qubit rotation operations and a two-qubit logic gate [20]. Single-qubit rotation operations are easy to be realized while two-qubit logic gates are complex to be implemented since it is difficult to obtain interaction between two qubits. Consequently, how to implement a two-qubit logic gate becomes an important and challenging topic in implementing quantum computation. Currently, for superconducting systems there are two conceptually different methods to obtain interaction between qubits. One is to employ direct interaction between two qubits through connecting them with capacitors, Josephson junctions, and dc-SQUID [2, 3, 21, 22, 23]. In this method, we only obtain the nearest neighbor interaction between qubits. Thus we can not couple selectively any two qubits in a large qubit network. The other method is to obtain effective interaction between two qubits by coupling them to a boson mode called data bus [24, 25, 26, 27, 28]. The second method can selectively couple any two qubits in a large qubit network. In practice, selective and controllable interaction between two qubits is expected in implementing quantum computation [29, 30]. Recently, some experiments have demonstrated interaction between two superconducting qubits [31, 32, 33, 34, 35]. It should be mentioned that, however, in these experiments the interactions between two qubits are only controllable but not selective. In this letter, we propose a SQC scheme to implement two-qubit logic gates with a controllable and selective interaction. In our scheme, we introduce a new type of data bus: a nanomechanical resonator (NAMR) [36, 37]. We show that a controllable interaction between qubits can be obtained by making use of the data bus, and a selective interaction between qubits can be realized when many qubits are involved.

The system under our consideration consists of two superconducting cooper-pair box (CPB) qubits fabricated by inserting a superconducting loop by two identical Josephson junctions as shown in Fig. 1. The two superconducting loops share a common circuit in which an NAMR is built. Two gate voltage sources V_{g_1} and V_{g_2} are used to control the two CPBs through corresponding gate capacitors. Moreover, the two CPBs can also be manipulated via external magnetic fluxes threading the loops. The physical mechanism of the interaction between the two CPBs can be explained as follows. The vibration of the NAMR changes the effective areas of two loops and the magnetic fluxes in the loops [38, 39]. When the area of the first loop increases (decreases), the area of the second loop decreases (increases). This correlative relation between the areas of two loops leads to an effective interaction between two qubits.

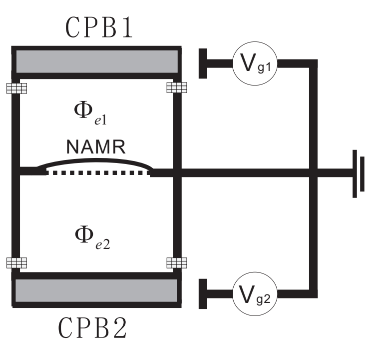

FIG. 1: Schematic diagram of the system of two CPBs coupled with an NAMR. Here V_{gk} and Φ_{ek} are gate voltages and external biasing fluxes, respectively.

The Hamiltonian of the total system [1] can be written

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as

$$H = \omega b^\dagger b + \sum_{k=1}^{2} 2E_{Ck}(2N_{gk}-1)\sigma_{zk} - \sum_{k=1}^{2} E_{Jk}\cos\left(\pi\frac{\Phi_{ek}}{\Phi_0}\right)\sigma_{xk}, \quad (1)$$

where ω is the flexural-mode frequency of the NAMR denoted by operators b and $b^\dagger$. $\Phi_0 = h/2e$ and Φ_{ek} are the flux quanta and bias flux of kth CPB, respectively. The single-electron charging energy is given by $E_{ck} = e^2/[2(2C_{Jk} + C_{gk})]$ where C_{Jk} and C_{gk} are the capacitance of each Josephson junction and the gate capacitance in the CPB k, respectively. The gate charge number is given by $N_{gk} = C_{gk}V_{gk}/2e$ with V_{gk} being gate voltage of kth CPB. The gate charge number can be controlled through the gate voltage. The Pauli operators used in Eq. (1) are defined by

$$\sigma_{zk} = |0\rangle_k\langle 0|_k - |1\rangle_k\langle 1|_k, \quad \sigma_{xk} = |0\rangle_k\langle 1|_k + |1\rangle_k\langle 0|_k, \quad (2)$$

where CPB's charge states $|0\rangle_k$ and $|1\rangle_k$ correspond to zero and one extra cooper-pair on the inland, respectively. Consistent with Fig. 1, we denote the upper and nether CPBs with indexes 1 and 2, respectively. We set $\hbar = 1$ throughout this paper.

The external magnetic fluxes in kth loop [38, 39] can be expressed as the sum of two terms,

$$\Phi_{ek} = \Phi_{bk} + (-1)^k BLx, \quad (3)$$

where the first term Φ_{bk} is the external magnetic fluxes in loop k when the NAMR does not vibrate while the second term describes contribution from the vibration of the NAMR with B being the magnetic field biasing on the two loops, L the effective length, and x the displacement of the NAMR. Assume that the mass of the NAMR is m and then we can write the displacement as $x = (b^\dagger + b)/\sqrt{2m\omega}$.

By substituting Eq. (3) into Eq. (1), we reduce the expression of Eq. (1) to the following form,

$$H = \omega b^\dagger b + \sum_{k=1}^{2} 2E_{ck}(2N_{gk}-1)\sigma_{zk} - \sum_{k=1}^{2} E_{Jk}\left[\cos\left(\pi\frac{\Phi_{bk}}{\Phi_0}\right)\cos\left(\pi\frac{BLx}{\Phi_0}\right) - (-1)^k\sin\left(\pi\frac{\Phi_{bk}}{\Phi_0}\right)\sin\left(\pi\frac{BLx}{\Phi_0}\right)\right]\sigma_{xk}. \quad (4)$$

It is straightforward to see that by controlling the biasing fluxes Φ_{bk} we can choose the sine or cosine parts in the Hamiltonian given by Eq. (4). In particular, when $\sin(\pi\Phi_{bk}/\Phi_0) = 1$, the Hamiltonian given by Eq. (4) becomes

$$H = \omega b^\dagger b + \sum_{i=1}^{2} 2E_{Ck}(2N_{gk}-1)\sigma_{zk} + \sum_{k=1}^{2}(-1)^k E_{Jk}\sin\left(\pi\frac{BLx}{\Phi_0}\right)\sigma_{xk}, \quad (5)$$

which can be reduced to the following form by expanding the sine function up to first-order in x,

$$H = \omega b^\dagger b + \sum_{k=1}^{2}\left[\frac{\omega_k}{2}\sigma_{zk} + (-1)^k g_k(b^\dagger + b)\sigma_{xk}\right], \quad (6)$$

where the effective energy separation and the coupling constant are given by

$$\omega_k = 4E_{Ck}(2N_{gk}-1), \quad g_k = E_{Jk}\pi BL/(\Phi_0\sqrt{2m\omega}). \quad (7)$$

The Hamiltonian (6) is well known in quantum optics since it is the same as the Hamiltonian of two two-level atoms interacting with a cavity field. Differently, the effective atomic energy separation in the present model can be controlled by tuning the gate voltage. For coherent manipulation of two interacting systems, one hopes to obtain a controllable interaction between the two systems. However, the coupling constant g_k in Eq. (6) is fixed for a given system, we can not turn on or off the coupling between the two CPBs and the NAMR on demand. In our present system, fortunately, we can obtain a controllable coupling between the two CPBs and the NAMR by replacing every Josephson junction in Fig. 1 by a SQUID-based superconducting loop [1]. Every loop contains two Josephson junctions with the same Josephson energy E^0_{Jk}. For kth CPB, there are three loops, left small one, right small one and middle big one. We assume that the fluxes Φ_{lk} and Φ_{rk} which bias respectively on left and right small loops have the same magnitude but opposite sign, i.e., $\Phi_{lk} = -\Phi_{rk} = \Phi_{xk}$. The coupling constant between kth CPB and the NAMR becomes

$$g'_k = 2E^0_{Jk}\cos(\pi\Phi_{xk}/\Phi_0)\pi BL/(\Phi_0\sqrt{2m\omega}), \quad (8)$$

which implies that we can control the coupling between the CPBs and the NAMR by tuning these biasing fluxes.

For realizing quantum computation, a set of universal quantum logic gates is necessary. A set of universal quantum logic gates consists of single-qubit logic operation and a nontrivial two-qubit logic gate such as CNOT and CP gate. In what follows we will show how to implement two-qubit logic gates between the two CPBs. From the Hamiltonian (6), we control the external fluxes and gate voltages such that only one kth CPB couples to the NAMR, the Hamiltonian becomes

$$H_I = \omega b^\dagger b + (-1)^k g'_k(b + b^\dagger)\sigma_{xk}. \quad (9)$$

In the interaction picture with respect to $H_0 = \omega b^\dagger b$, the Hamiltonian given by Eq. (9) is transformed to the following expression,

$$H'_k(t_k) = (-1)^k g'_k(be^{-i\omega t_k} + b^\dagger e^{i\omega t_k})\sigma_{xk}, \quad (10)$$

which leads to a unitary evolution operator,

$$U(t_k) = \overleftarrow{T} \exp\left[-i\int_0^{t_k} ds H_k'(s)\right], \qquad (11)$$

where $\overleftarrow{T}$ is the time-ordering operator. Up to a trivial phase factor this unitary evolution operator can be written as [40]

$$V(\alpha_k\sigma_{xk}) = \exp[b^\dagger\alpha_k\sigma_{xk} - b\alpha_k^*\sigma_{xk}], \qquad (12)$$

where we have introduced the coupling parameter between kth qubit and the NAMR

$$\alpha_k = (-1)^k g_k' \left(1 - e^{i\omega t_k}\right)/\omega, \qquad (13)$$

which can be controlled with external flux Φ_{xk} and evolution time t_k.

In what follows we show how to realize two-qubit gate by using the unitary evolution given by Eq. (12). We note that the evolution operator given by Eq. (12) is a controlled displacement operator [41], which can produce a displacement of the NAMR, conditioned on eigenstates of the operator σ_{xk}. It has been shown that, using the proper controlled-displacement operators we can implement two-qubit gates. The sequence of operations is arranged as follows,

$$\begin{aligned} U(\alpha_1,\alpha_2) &= V(\alpha_2\sigma_{x2}) \otimes V(\alpha_1\sigma_{x1})V(-\alpha_2\sigma_{x2}) \\ &\quad \otimes V(-\alpha_1\sigma_{x1}), \end{aligned} \qquad (14)$$

which can be written as

$$U(\alpha_1,\alpha_2) = \exp[i\theta\sigma_{x1}\sigma_{x2}], \qquad (15)$$

where the effective coupling constant

$$\theta = 2|\alpha_1||\alpha_2|\sin(\varphi_2 - \varphi_1) \qquad (16)$$

with $\alpha_k = |\alpha_k|e^{i\varphi_k}$. Because the coupling constant is a function of α_k, according to the expressions of α_k and g_k' we can see that it is possible to control this coupling constant by tuning the external fluxes Φ_{xk} and controlling the evolution times t_k. Choosing proper parameters such that $\theta = \pi/4$, the operator given by Eq. (15) is equivalent to a nontrivial two-qubit gate [42, 43, 44].

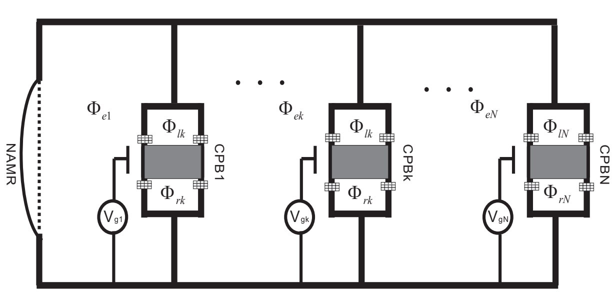

FIG. 2: Schematic diagram of a multi-qubit superconducting circuit in which N qubits couple with an NAMR.

In practice, for performing a certain quantum computation mission, we need to operate many qubits during the operation process. Hence how to integrate the present model to multi-qubit circuit and how to obtain selective and controllable two-qubit logic gates are main missions. Fortunately, the integration of multi-qubit circuits is straightforward. We plot the schematic diagram of our arrangement in Fig. 2. The coupling of the Josephson charge qubit k with the NAMR can be controlled by tuning the biasing fluxes Φ_{lk} and Φ_{rk}. For realizing a two-qubit logic gate between qubits i and j, we couple the qubits i and j with the NAMR and decouple other qubits with the NAMR by controlling the biasing fluxes. Moreover, to obtain the required Hamiltonian given by Eq. (6), we can control the biasing magnetic field threading these loops to meet the condition $\sin(\pi\Phi_{bk}/\Phi_0) = 1$.

In conclusion, we have proposed a scheme to implement two-qubit logic gates in two CPBs by coupling them with an NAMR. Under different work conditions, we have shown that we can realize two-qubit logic gates in three cases. In the first case, we carefully tune the gate voltage and these biasing fluxes of the CPBs to obtain controlled displacement operations between the CPBs and the NAMR. And a certain sequence of controlled displacement operations can reduce to a type of $\sigma_{x1}\sigma_{x2}$ interaction between the two CPBs. In the second case, we decouple the coupling between the CPBs and the NAMR and also obtain a type of $\sigma_{x1}\sigma_{x2}$ interaction between the two CPBs at some selective time points. In the third method, we tune carefully the energy separations of the CPBs such that the NAMR is large detuning from the two CPBs. Under the large detuning condition, we eliminate adiabatically the NAMR and obtain a type of $(\sigma_{+1}\sigma_{-2}+\sigma_{-1}\sigma_{+2})$ interaction between the two CPBs.

Finally, we give an estimation of the experimental feasibility for the present scheme. In our scheme, we should consider the following two time scales: the time required for implementing a two-qubit logic gate and the lifetime of the qubits. We set the following parameters [38], $B \approx 0.1$ T, $l \approx 30$ μm, $x_0 \approx 5\times10^{-13}$ m, $E_J^0 \approx 5$ GHz. So we can obtain the maximum coupling constant $g_{max}' \approx 30$ MHz. We choose a type of NAMR, $\omega \approx 2\pi \times 100$ MHz, and quality factor $Q \approx 10^5$. For simplicity, we assume that the two qubits have the same parameters. In the first case, for meeting the condition $|\alpha_1||\alpha_2|\sin(\varphi_2 - \varphi_1) = \pi/4$, we need to control the time t_k of interaction between the qubit k and the NAMR to meet the equation $4g_1'g_2'\sin(\omega t_1/2)\sin(\omega t_2/2)\sin(\omega(t_1 - t_2)/2)/\omega^2 = \pi/4$. Using the above parameters, this equation reduces to $\sin(\omega t_1/2)\sin(\omega t_2/2)\sin(\omega(t_1 - t_2)/2) = 0.69$, so the fastest gate implementation needs $t_{tot} \sim 10^{-7}$ s. In the second case, $\theta = 4n\pi g_1'g_2'/\omega^2 \sim \pi/4$ corresponds to $t \sim 10^{-7}$ s. Corresponding to the third case, we set the detuning $\Delta \approx 5g$, then the time required for implementing a $\sqrt{i\mathtt{SWAP}}$ gate $t = \Delta\pi/(4g^2) \approx 1\times10^{-7}$ s. Moreover, for a type of CPB, the dissipation time $T_1 \approx 1 \sim 10$ μs and the dephasing time $T_2 \approx 0.1 \sim 1$ μs [3]. Therefore, in our present scheme the time required for implementing

two-qubit logic gates is shorter than the lifetime of the qubit.

Acknowledgments

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